

Phenomenology of spin 3/2 baryons with pentaquarks

Tetsuo Hyodo* and Atsushi Hosaka

Research Center for Nuclear Physics (RCNP), Ibaraki, Osaka 567-0047, Japan

(Dated: February 7, 2008)

We examine several assignments of spin and parity for the pentaquark Θ^+ state ($J^P = 1/2^\pm, 3/2^\pm$) in connection with known baryon resonances. Assuming that the Θ^+ belongs to an antidecuplet representation which mixes with an octet, we calculate the mass spectra of the flavor partners of the Θ^+ based on the SU(3) symmetry. The decay widths of the Θ^+ and nucleon partners are analyzed for the consistency check of the mixing angle obtained from the masses. It is found that a suitable choice of the mixing angle successfully reproduces the observed masses of $\Theta(1540)$ and $\Xi_{3/2}(1860)$, when their spin and parity are assigned to be $J^P = 3/2^-$, together with other $J^P = 3/2^-$ resonances. The decay widths of $\Theta \rightarrow KN$, $N(1520) \rightarrow \pi N$, and $N(1700) \rightarrow \pi N$ are also reproduced simultaneously.

PACS numbers: 14.20.-c, 11.30.Hv, 12.40.Yx, 13.30.Eg

I. INTRODUCTION

In recent years, there has been a remarkable development in hadron spectroscopy. One of the most interesting observations is the evidence of the exotic baryon Θ^+ , reported first by the LEPS collaboration [1]. Subsequently, a signal of another exotic state Ξ^{--} was observed [2]. The spin and parity of Θ^+ and Ξ^{--} are not yet determined experimentally. Since these states can be constructed minimally with five valence quarks, they are called pentaquarks. Evidences of the exotic pentaquarks have been stimulating many theoretical studies [3, 4, 5, 6, 7, 8, 9].

In the study of these exotic particles, it should be important to identify other members with nonexotic flavors in the same SU(3) multiplet which the exotic particles belong to. This is naturally expected from the successes of SU(3) flavor symmetry with its breaking in hadron masses and interactions [10]. In other words, the existence of exotic particles would require the flavor partners, if the flavor SU(3) symmetry plays the same role as in the ordinary three-quark baryons.

An interesting proposal was made by Jaffe and Wilczek [11], based on the assumption of the strong diquark correlation in hadrons and the representation mixing of an octet (**8**) with an antidecuplet ($\overline{\mathbf{10}}$). The attractive diquark correlation in the scalar-isoscalar channel leads to the spin and parity $J^P = 1/2^+$ for the Θ^+ . With the ideal mixing of **8** and $\overline{\mathbf{10}}$, in which states are classified by the number of strange and antistrange quarks, $N(1710)$ and $N(1440)$ resonances are well fit as members of the multiplet together with the Θ^+ . However, it was pointed out that mixing angles close to the ideal one encountered a problem in the decay pattern of $N(1710) \rightarrow \pi N$ and $N(1440) \rightarrow \pi N$. Rather, their decays implied a small mixing angle [12, 13, 14]. This is intuitively understood by observing the broad decay width of $N(1440) \rightarrow \pi N$ and the narrow widths of

$N(1710) \rightarrow \pi N$ and $\Theta \rightarrow KN$ [12].

Employing the **8**- $\overline{\mathbf{10}}$ mixing scenario, here we examine the possibilities to assign other quantum numbers, such as $1/2^-$, $3/2^+$, $3/2^-$, and search the nucleon partners among the known resonances. For convenience, properties of relevant resonances are summarized in Appendix A.

The present study is based on the flavor SU(3) symmetry, experimental mass spectra and decay widths of the Θ^+ , the Ξ^{--} and known baryon resonances. Hence, our analysis presented here is phenomenological, but does not rely upon any specific models. For instance, we do not have to specify the quark contents of the baryons. Although the exotic states require minimally five quarks, nonexotic partners do not have to. Instead, we expect that the resulting properties such as masses and decay rates reflect information from which we hope to learn internal structure of the baryons.

II. ANALYSIS WITH PURE ANTIDECUPLET

First we briefly discuss the case where the Θ^+ belongs to the pure $\overline{\mathbf{10}}$ without mixing with other representations. In this case, the masses of particles belonging to the $\overline{\mathbf{10}}$ can be determined by the Gell-Mann–Okubo (GMO) mass formula with equal splitting

$$M(\overline{\mathbf{10}}; Y) \equiv \langle \overline{\mathbf{10}}; Y | \mathcal{H} | \overline{\mathbf{10}}; Y \rangle = M_{\overline{\mathbf{10}}} - aY, \quad (1)$$

where Y is the hypercharge of the state, and \mathcal{H} denotes the mass matrix. Note that at this point the spin and parity J^P are not yet specified. This will be assigned as explained below.

In Eq. (1), there are two parameters, $M_{\overline{\mathbf{10}}}$ and a , which are not determined by the flavor SU(3) symmetry. However, we can estimate the order of these parameters by considering their physical meanings. For instance, in a constituent quark model, $\overline{\mathbf{10}}$ can be minimally expressed as four quarks and one antiquark. Therefore, $M_{\overline{\mathbf{10}}}$ should be larger than the masses of three-quark baryons, such as the lowest-lying octet baryons. In this picture, the mass

*Electronic address: hyodo@rcnp.osaka-u.ac.jp

difference of $\Xi(ssqq\bar{q})$ and $\Theta(qqq\bar{s})$, namely $3a$, should be the constituent mass difference of the s and the ud quarks, which is about 100-250 MeV [15]. On the other hand, in the chiral quark soliton model, $3a$ is related to the pion nucleon sigma term [16]. In this picture $3a$ can take values in the range of 300-400 MeV, due to the experimental uncertainty of the pion nucleon sigma term $\Sigma_{\pi N} = 64\text{--}79$ MeV [17, 18, 19]. Note that in the chiral quark model, spin and parity are assigned as $J^P = 1/2^+$ for the antidecuplet.

Taking into account the above estimation, we test several parameter sets fixed by the experimentally known masses of particles. The results are summarized in Table I. First, we determine the parameters by accommodating $\Theta(1540)$ and $\Xi(1860)$ in the multiplet. In this case we obtain the mass of the N and Σ states at 1647 and 1753 MeV, respectively. Since these values are close to the masses of the $1/2^-$ baryons $N(1650)$ and $\Sigma(1750)$, we expect their spin and parity to be $J^P = 1/2^-$. For $J^P = 1/2^+$, we adopt the $N(1710)$ as the nucleon partner, and predict the Σ and Ξ states. This assignment corresponds to the original assignment of the prediction [20]. For $J^P = 3/2^+$, we pick up $N(1720)$, and for $J^P = 3/2^-$, $N(1700)$. In the three cases of $J^P = 1/2^+, 3/2^\pm$, the exotic Ξ resonance is predicted to be higher than 2 GeV, and the inclusion of $\Xi(1860)$ in the same multiplet seems to be difficult. Furthermore, the Σ states around 1.8-1.9 GeV are not well assigned (either two-star for $J^P = 1/2^+$, or not seen for $J^P = 3/2^\pm$). Therefore, fitting the masses in the pure antidecuplet scheme seems to favor $J^P = 1/2^-$.

Next we study the decay width of the N^* resonances with the above assignments. For the decay of a resonance R , we define the dimensionless coupling constant g_R by

$$\Gamma_R \equiv g_R^2 F_I \frac{p^{2l+1}}{M_R^{2l}}, \quad (2)$$

where p is the relative three momentum of the decaying particles in the resonance rest frame, Γ_R and M_R are the decay width and the mass of the resonance R . F_I is the isospin factor, which takes the value 2 for $\Theta \rightarrow KN$ and 3 for $N^* \rightarrow \pi N$. Assuming flavor $SU(3)$ symmetry, a relation between the coupling constants of $\Theta \rightarrow KN$ and $N^* \rightarrow \pi N$ is given by:

$$g_{\Theta KN} = \sqrt{6} g_{N^* \pi N}. \quad (3)$$

Here we adopt the definition of the coupling constant in Ref. [21]. Note that this definition is different from Refs. [12, 13], in which $g \equiv \sqrt{g_R^2 F_I}$ is used. With these formulae (2) and (3), we calculate the decay width of the Θ^+ from those of $N^* \rightarrow \pi N$ of the nucleon partner. Results are also shown in Table I. We quote the errors coming from experimental uncertainties in the total decay widths and branching ratios, taken from the Particle Data Group [22]. It is easily seen that as the partial wave of the two-body final states becomes higher, the decay width of the resonance becomes narrower, due to the

TABLE I: Summary of section II. Mass spectra and Θ^+ decay width are shown for several assignments of quantum numbers. For $1/2^-$ the masses of Θ and Ξ are the input parameters, while for $1/2^+, 3/2^\pm$, the masses of Θ and N are the input parameters. Values in parenthesis are the predictions, and we show the candidates to be assigned for the states. All values are listed in units of MeV.

J^P	M_Θ	M_N	M_Σ	M_Ξ	Γ_Θ
$1/2^-$	1540	[1647]	[1753]	1860	$156.1^{+90.8}_{-73.3}$
		$N(1650)$	$\Sigma(1750)$		
$1/2^+$	1540	1710	[1880]	[2050]	$7.2^{+15.3}_{-4.6}$
			$\Sigma(1880)$	$\Xi(2030)$	
$3/2^+$	1540	1720	[1900]	[2080]	$10.6^{+7.0}_{-5.0}$
			-	-	
$3/2^-$	1540	1700	[1860]	[2020]	$1.3^{+1.2}_{-0.9}$
			-	$\Xi(2030)$	

effect of the centrifugal barrier. Considering the experimental width of the Θ^+ , the results of $J^P = 3/2^-, 3/2^+$, $1/2^+$ are acceptable, but the result of the $J^P = 1/2^-$ case, which is of the order of hundred MeV, is unrealistic.

In summary, it seems difficult to regard the Θ^+ as a member of the pure antidecuplet $\overline{\mathbf{10}}$ together with known resonances of $J^P = 1/2^\pm, 3/2^\pm$, in fitting both their masses and decay widths.

III. ANALYSIS WITH OCTET-ANTIDECUPLET MIXING

In this section we consider the representation mixing between $\overline{\mathbf{10}}$ and $\mathbf{8}$. In principle, it is possible to take into account the mixing with multiplets of higher dimension, such as $\mathbf{27}$ and $\mathbf{35}$. However, particles in such higher representations will have heavier masses. Furthermore, the higher representations bring more states with exotic quantum numbers, which are not controlled by the known experimental information. Here we work under the assumption of minimal $\mathbf{8}\text{--}\overline{\mathbf{10}}$ mixing. Also we do not consider the possible mixing with other octets, such as ground states [23].

The nucleon and Σ states in the $\mathbf{8}$ will mix with the states in the $\overline{\mathbf{10}}$ of the same quantum numbers. Denoting the mixing angles of the N and the Σ as θ_N and θ_Σ , the physical states are represented as

$$\begin{aligned} |N_1\rangle &= |\mathbf{8}, N\rangle \cos \theta_N - |\overline{\mathbf{10}}, N\rangle \sin \theta_N, \\ |N_2\rangle &= |\overline{\mathbf{10}}, N\rangle \cos \theta_N + |\mathbf{8}, N\rangle \sin \theta_N, \end{aligned} \quad (4)$$

and

$$\begin{aligned} |\Sigma_1\rangle &= |\mathbf{8}, \Sigma\rangle \cos \theta_\Sigma - |\overline{\mathbf{10}}, \Sigma\rangle \sin \theta_\Sigma, \\ |\Sigma_2\rangle &= |\overline{\mathbf{10}}, \Sigma\rangle \cos \theta_\Sigma + |\mathbf{8}, \Sigma\rangle \sin \theta_\Sigma. \end{aligned} \quad (5)$$

To avoid redundant duplication, the domain of the mixing angles is restricted in $0 \leq \theta < \pi/2$, and we will find

solutions for N_1 and Σ_1 lighter than N_2 and Σ_2 , respectively. The reason for these restrictions is explained in Appendix B.

When we construct $\overline{10}$ and $\mathbf{8}$ from five quarks, the eigenvalues of the strange quark (antiquark) number operator n_s of nucleon states become fractional. In the scenario of the ideal mixing of Jaffe and Wilczek, the physical states are given as

$$|N_1\rangle = \sqrt{\frac{2}{3}}|\mathbf{8}, N\rangle - \sqrt{\frac{1}{3}}|\overline{10}, N\rangle, \quad (6)$$

$$|N_2\rangle = \sqrt{\frac{2}{3}}|\overline{10}, N\rangle + \sqrt{\frac{1}{3}}|\mathbf{8}, N\rangle, \quad (7)$$

such that $\langle N_1 | n_s | N_1 \rangle = 0$ and $\langle N_2 | n_s | N_2 \rangle = 2$. In this case, the mixing angle is

$$\theta_N \sim 35.2^\circ. \quad (8)$$

This value will be compared with the angle obtained from the mass spectrum of known resonances. In the Jaffe-Wilczek model [11], $N(1440)$ and $N(1710)$ are assigned to N_1 and N_2 , respectively. Notice that the separation of the $s\bar{s}$ component in the ideal mixing is only meaningful for mixing between five-quark states, while the number of quarks in the baryons is arbitrary in the present general framework.

It is worth mentioning that the mixing angle θ_N for $1/2^+$ case is calculated through the dynamical study of constituent quark model [24]. The resulting value is $\theta_N \sim 35.34^\circ$, which is very close to the ideal mixing angle (8).

A. Mass spectrum

Let us start with the GMO mass formulae for $\overline{10}$ and $\mathbf{8}$:

$$M(\overline{10}; Y) \equiv \langle \overline{10}; Y | \mathcal{H} | \overline{10}; Y \rangle = M_{\overline{10}} - aY, \quad (9)$$

$$\begin{aligned} M(\mathbf{8}; I, Y) &\equiv \langle \mathbf{8}; I, Y | \mathcal{H} | \mathbf{8}; I, Y \rangle \\ &= M_{\mathbf{8}} - bY + c \left[I(I+1) - \frac{1}{4}Y^2 \right], \end{aligned} \quad (10)$$

where Y and I are the hypercharge and the isospin of the state. Under representation mixing as in Eqs. (4) and (5), the two nucleons ($N_{\mathbf{8}}, N_{\overline{10}}$) and the two sigma states ($\Sigma_{\mathbf{8}}, \Sigma_{\overline{10}}$) mix, and their mass matrices are given by 2×2 matrices. The diagonal components are given by Eqs. (9) and (10), while the off-diagonal elements are given as

$$\langle \mathbf{8}, N | \mathcal{H} | \overline{10}, N \rangle = \langle \mathbf{8}, \Sigma | \mathcal{H} | \overline{10}, \Sigma \rangle \equiv \delta. \quad (11)$$

The equivalence of the two off-diagonal elements can be verified when the symmetry breaking term is given by λ_8 due to the large strange quark mass [17].

The physical states $|N_i\rangle$ and $|\Sigma_i\rangle$ diagonalize \mathcal{H} . Therefore, we have the relations

$$\tan 2\theta_N = \frac{2\delta}{M_{\overline{10}} - M_{\mathbf{8}} - a + b - \frac{1}{2}c}, \quad (12)$$

and

$$\tan 2\theta_\Sigma = \frac{2\delta}{M_{\overline{10}} - M_{\mathbf{8}} - 2c}. \quad (13)$$

Now we have the mass formulae for the states

$$M_\Theta = M_{\overline{10}} - 2a, \quad (14)$$

$$\begin{aligned} M_{N_1} &= \left(M_{\mathbf{8}} - b + \frac{1}{2}c \right) \cos^2 \theta_N + (M_{\overline{10}} - a) \sin^2 \theta_N \\ &\quad - \delta \sin 2\theta_N, \end{aligned} \quad (15)$$

$$\begin{aligned} M_{N_2} &= \left(M_{\mathbf{8}} - b + \frac{1}{2}c \right) \sin^2 \theta_N + (M_{\overline{10}} - a) \cos^2 \theta_N \\ &\quad + \delta \sin 2\theta_N, \end{aligned} \quad (16)$$

$$M_{\Sigma_1} = (M_{\mathbf{8}} + 2c) \cos^2 \theta_\Sigma + M_{\overline{10}} \sin^2 \theta_\Sigma - \delta \sin 2\theta_\Sigma, \quad (17)$$

$$M_{\Sigma_2} = (M_{\mathbf{8}} + 2c) \sin^2 \theta_\Sigma + M_{\overline{10}} \cos^2 \theta_\Sigma + \delta \sin 2\theta_\Sigma, \quad (18)$$

$$M_\Lambda = M_{\mathbf{8}}, \quad (19)$$

$$M_{\Xi_8} = M_{\mathbf{8}} + b + \frac{1}{2}c, \quad (20)$$

$$M_{\Xi_{\overline{10}}} = M_{\overline{10}} + a. \quad (21)$$

We have altogether six parameters $M_{\mathbf{8}}, M_{\overline{10}}, a, b, c$ and δ .

Let us first examine the case of $J^P = 1/2^+$ [13]. Possible candidates for the partners of the exotic states $\Theta(1540)$ and $\Xi_{\overline{10}}(1860)$ are the following:

$$\begin{aligned} &N(1440), N(1710), \\ &\Lambda(1600), \\ &\Sigma(1660), \Sigma(1880). \end{aligned}$$

In order to fix the six parameters, we need to assign six particles as input. Using $\Theta(1540)$, $N_1(1440)$, $N_2(1710)$, $\Lambda(1600)$, $\Sigma_1(1660)$, $\Xi_{\overline{10}}(1860)$, we obtain the parameters as given in Table II. The resulting mass spectrum together with the two predicted masses, $\Sigma_1 = 1894$ MeV and $\Xi_8 = 1797$ MeV, are given in Table III and also shown in the left panel of Fig. 1. For reference, in Table II and III we show the parameters and masses of Ref. [13], in which all known resonances including $\Sigma(1660)$ and $\Sigma(1880)$ are used to perform the χ^2 fitting. In Fig. 1, the spectra from experiment and those before the representation mixing are also plotted.

As we see in Table III and Fig. 1, without using the Σ_2 for the fitting, this state appears in the proper position to be assigned as $\Sigma(1880)$. Considering the experimental uncertainty in the masses, these two parameter sets (the one determined in this work and the one in Ref. [13]) can be regarded as the same one. In both cases, we need a new Ξ state around 1790-1800 MeV, but the overall description of the mass spectrum is acceptable. Note that the mixing angle $\theta_N \sim 30^\circ$ is compatible with the one of the ideal mixing (8), if we consider the experimental uncertainty of masses [13].

TABLE II: Parameters for $1/2^+$ case. All values are listed in MeV except for the mixing angles.

	M_8	$M_{\overline{10}}$	a	b	c	δ	θ_N	θ_Σ
This work	1600	1753.3	106.7	146.7	100.1	114.4	29.0°	50.8°
Ref. [13]	1600	1755	107	144	93	123	29.7°	41.4°

TABLE III: Mass spectra for $1/2^+$ case. All values are listed in MeV. Values in parenthesis (Σ_2 and Ξ_8 of Set 1, Ξ_8 of Ref. [13]) are predictions (those which are not used in the fitting).

	Θ	N_1	N_2	Σ_1	Σ_2	Λ	Ξ_8	$\Xi_{\overline{10}}$
This work	1540	1440	1710	1660	[1894]	1600	[1797]	1860
Ref. [13]	1541	1432	1718	1650	1891	1600	[1791]	1862

It is interesting to observe that in the spectrum of the octet, as shown in Fig. 1, the Ξ_8 and the Σ_8 are almost degenerate, reflecting the large value for the parameter $c \sim 100$ MeV, which is responsible for the splitting of Λ and Σ . For the ground state octet, Eq. (10) is well satisfied with $b = 139.3$ MeV and $c = 40.2$ MeV [17]. This point will be discussed later.

Now we examine the other cases of J^P . For $J^P = 1/2^-$, as we observed in the previous section, the pure $\overline{10}$ assignment works well for the mass spectrum, which implies that the mixing with 8 is small, as long as we adopt $N(1650)$ and $\Sigma(1750)$ in the multiplet. Then the results of $1/2^-$ with the mixing do not change from the previous results of the pure $\overline{10}$ assignment, which eventually lead to a broad width of $\Theta^+ \rightarrow KN$ of order 100 MeV. Hence, it is not realistic to assign $1/2^-$, even if we consider the representation mixing.

Next we consider the $3/2^+$ case. In this case candidate states are not well established. As we see in Appendix A, no state exists for Σ and Ξ , except for two- or one-star resonances. Furthermore, the states are distributed in a wide energy range, and sometimes it is not possible to assign these particles in the $8-\overline{10}$ representation scheme. For instance, if we choose $N(1720)$, $N(1900)$, $\Lambda(1890)$, $\Sigma(1840)$ and exotic states, no solution is found for the mixing angle. Therefore, at this moment, it is not meaningful to study the $3/2^+$ case unless more states with $3/2^+$ will be observed.

Now we look at the $3/2^-$ case. In contrast to the $3/2^+$ case, there are several well-established resonances. Possible candidates are

$$\begin{aligned} &N(1520), N(1700), \\ &\Lambda(1520), \Lambda(1690), \\ &\Sigma(1670), \Sigma(1940), \\ &\Xi(1820). \end{aligned}$$

Following the same procedure as before, we first choose the following four resonances as inputs: $\Theta(1540)$, $N_1(1700)$, $N_2(1520)$, and $\Xi_{3/2}(1860)$. For the remaining

two to determine the six parameters, we examine four different choices of Σ and Λ states;

$$\begin{aligned} &\Sigma(1670) \text{ and } \Lambda(1690) \text{ (set1),} \\ &\Sigma(1670) \text{ and } \Lambda(1520) \text{ (set2),} \\ &\Sigma(1940) \text{ and } \Lambda(1690) \text{ (set3),} \\ &\Sigma(1940) \text{ and } \Lambda(1520) \text{ (set4).} \end{aligned} \quad (22)$$

We have obtained the parameters as given in Table IV, and predicted masses of other members are shown in Table V. The masses of $N(1520)$ and $N(1700)$ determine the mixing angle of nucleons $\theta_N \sim 33^\circ$, which is close to the ideal one. In the parameter sets 1 and 2 (sets 3 and 4), the $\Sigma(1670)$ state of a lower mass (the $\Sigma(1940)$ state of a higher mass) is chosen but with different Λ 's, $\Lambda(1690)$ and $\Lambda(1520)$. Accordingly, they predict the higher $\Sigma(1834)$ state (the lower $\Sigma(1717)$ state) with the mixing angle $\theta_\Sigma = 44.6^\circ (= 66.2^\circ)$. Interestingly, parameters of set 1 provide $M_{\Xi_8} \sim 1837$ MeV, which is close to the known three-star resonance $\Xi(1820)$ of $J^P = 3/2^-$. Parameters of set 4 predict $M_{\Xi_8} \sim 1659$ MeV, which is close to another known resonance $\Xi(1690)$. Since the J^P of this state is not known, this fitting scheme predicts J^P of $\Xi(1690)$ to be $3/2^-$. In these two cases, we have obtained acceptable assignments, especially for set 1, although a new Σ state is necessary to complete the multiplet in both cases. The spectrum of set 1 is also shown in Fig. 1.

Let us briefly look at the octet and antidecuplet spectra of $1/2^+$ and $3/2^-$ resonances as shown in Fig. 1. The antidecuplet spectrum is simple, since the GMO mass formula contains only one parameter which describes the size of the splitting. Contrarily, the octet spectrum contains two parameters which could reflect more information on different internal structure. As mentioned before, in the octet spectrum of $1/2^+$, the mass of Σ_8 is pushed up slightly above Ξ_8 , significantly higher than Λ_8 . This pattern resembles the octet spectrum which is obtained in the Jaffe-Wilczek model, where the baryons are made with two flavor $\overline{3}$ diquarks and one antiquark. In contrast, the spectrum of the octet of $3/2^-$ resembles the one of the ground state octet, what is reflected in the parameters $(b, c) = (131.9, 30.5)$ MeV, close to $(b, c) = (139.3, 40.2)$ MeV for the ground states. This is not far from the prediction of an additive quark model of three valence quarks. It would be interesting to investigate further the quark contents from such a different pattern of the mass spectrum.

B. Decay width

Here we study the consistency of the mixing angle obtained from mass spectra and the one obtained from nucleon decay widths. Using Eq. (3), we define a universal coupling constant $g_{\overline{10}}$ as

$$g_{\Theta KN} = \sqrt{6} g_{N_{\overline{10}} \pi N} \equiv g_{\overline{10}}. \quad (23)$$

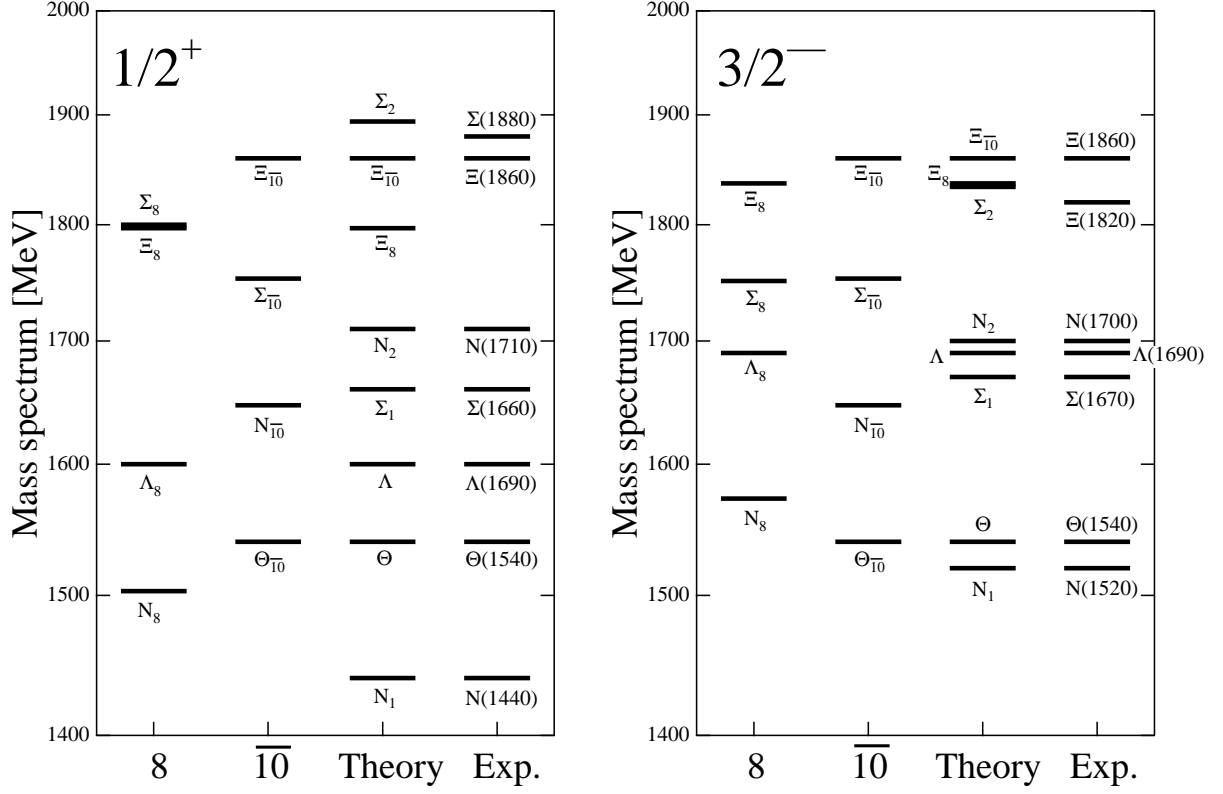


FIG. 1: Results of mass spectra with representation mixing. Theoretical masses of the octet, antidecuplet, and the one with mixing are compared with the experimental masses. In the left panel, we show the results with $J^P = 1/2^+$, while the results with $J^P = 3/2^-$ (set 1) are presented in the right panel.

TABLE IV: Parameters for $3/2^-$ case. All values are listed in MeV except for the mixing angles.

	M_8	$M_{\bar{10}}$	a	b	c	δ	θ_N	θ_Σ
set 1	1690	1753.3	106.7	131.9	30.5	82.2	33.0°	44.6°
set 2	1520	1753.3	106.7	4.4	115.5	82.2	33.0°	44.6°
set 3	1690	1753.3	106.7	170.1	106.9	82.2	33.0°	66.2°
set 4	1520	1753.3	106.7	42.6	191.9	82.2	33.0°	66.2°

TABLE V: Mass spectra for $3/2^-$ case. All values are listed in MeV. Values in parenthesis are predictions (those which are not used in the fitting).

	Θ	N_1	N_2	Σ_1	Σ_2	Λ	Ξ_8	$\Xi_{\bar{10}}$
set 1	1540	1520	1700	1670	[1834]	1690	[1837]	1860
set 2	1540	1520	1700	1670	[1834]	1520	[1582]	1860
set 3	1540	1520	1700	[1717]	1940	1690	[1914]	1860
set 4	1540	1520	1700	[1717]	1940	1520	[1659]	1860

The coupling constants of the πN decay modes from the N_8 , N_1 , and N_2 are defined as g_{N_8} , g_{N_1} , and g_{N_2} , respectively. The coupling constants of the physical nucleons

N_1 and N_2 are

$$g_{N_1} = g_{N_8} \cos \theta_N - \frac{g_{\bar{10}}}{\sqrt{6}} \sin \theta_N, \quad (24)$$

$$g_{N_2} = \frac{g_{\bar{10}}}{\sqrt{6}} \cos \theta_N + g_{N_8} \sin \theta_N, \quad (25)$$

which are related to the decay widths through Eq. (2). However, we cannot fix the relative phase between g_{N_8} and $g_{\bar{10}}$. Hence, there are two possibilities of mixing angles both of which reproduce the same decay widths. In Refs. [13, 14], one mixing angle is determined by neglecting $g_{\bar{10}}$ in Eqs. (24) and (25), which is considered to be small due to the narrow width of Θ^+ . Here we include the effect of $g_{\bar{10}}$ explicitly.

Let us examine the two cases, $1/2^+$ and $3/2^-$, in which we have obtained reasonable mass spectra. The data for decay widths and branching ratios to the πN channel of relevant nucleon resonances are shown in Table VI. Using the mixing angle determined from the mass spectrum and experimental information of $N^* \rightarrow \pi N$ decays, we obtain the decay width of the Θ^+ as shown in Table VII. The widths calculated with the ideal mixing angle are also presented for reference. Among the two values, the former corresponds to the same signs of g_{N_8} and $g_{\bar{10}}$ (phase

TABLE VI: Experimental data for the decay of N^* resonances. Values in parenthesis are the central values quoted in PDG [22].

J^P	Resonance	Γ_{tot} [MeV]	Fraction ($\Gamma_{\pi N}/\Gamma_{tot}$)
$1/2^+$	N(1440)	250-450 (350)	60-70 (65) %
	N(1710)	50-250 (100)	10-20 (15) %
$3/2^-$	N(1520)	110-135 (120)	50-60 (55) %
	N(1700)	50-150 (100)	5-15 (10) %

TABLE VII: Decay width of Θ^+ determined from the nucleon decays and the mixing angle obtained from the mass spectra. Phase 1 corresponds to the same signs of g_{N_8} and $g_{\overline{10}}$, while phase 2 corresponds to the opposite signs. All values are listed in MeV.

J^P	θ_N	Phase 1	Phase 2
$1/2^+$	29° (Mass)	29.1	103.3
	35.2° (Ideal)	49.3	131.8
$3/2^-$	33° (Mass)	3.1	20.0
	35.2° (Ideal)	3.9	21.3

1), while the latter to the opposite signs (phase 2).

For the $1/2^+$ case, the width is about 30 MeV when the mixing angle is determined by the mass spectrum, while about 50 MeV for the ideal mixing angle. Both values exceed the upper bound of the experimentally observed width. In contrast, the case $3/2^-$ predicts much narrower widths of the order of a few MeV both for the two mixing angles, which are compatible with the experimental upper bound of the Θ^+ width.

Alternatively, we can determine θ_N using the experimental decay widths of $\Theta \rightarrow KN$, $N_1 \rightarrow \pi N$ and $N_2 \rightarrow \pi N$. Here we choose the decay width of Θ^+ as 1 MeV. Using the central values of the decay widths of N(1440) and N(1710) and the experimental uncertainty, we obtain the nucleon mixing angle for the $1/2^+$ case

$$\begin{aligned}\theta_N &= 6^\circ \begin{smallmatrix} +9^\circ \\ -4^\circ \end{smallmatrix}, \\ \theta_N &= 14^\circ \begin{smallmatrix} +10^\circ \\ -4^\circ \end{smallmatrix},\end{aligned}\quad (26)$$

where the former corresponds to the phase 1 and the latter to the phase 2. On the other hand, with N(1520) and N(1700), the mixing angle for the $3/2^-$ case is

$$\begin{aligned}\theta_N &= 9^\circ \begin{smallmatrix} +9^\circ \\ -8^\circ \end{smallmatrix}, \\ \theta_N &= 24^\circ \begin{smallmatrix} +9^\circ \\ -9^\circ \end{smallmatrix}.\end{aligned}\quad (27)$$

For the case of $1/2^+$, the mixing angle of Eq. (26) may be compared with $\theta_N \sim 30^\circ$, which is determined from the fitting to the masses. If we consider the large uncertainty of the πN decay width of N(1440), the mixing angle (26) can be 24° , which is not very far from the angle determined by the masses $\theta_N \sim 30^\circ$. On the other hand, for the case of $3/2^-$, the mixing angle (27) agrees well with the angle determined by the masses $\theta_N \sim 33^\circ$. Considering the agreement of mixing angles and the relatively

small uncertainties in the experimental decay widths, the results with the $3/2^-$ case are favorable in the present fitting analysis.

IV. SUMMARY AND DISCUSSION

We have studied the mass spectra and decay widths of the baryons belonging to the **8** and $\overline{\mathbf{10}}$ based on the flavor SU(3) symmetry. As pointed out previously, it was confirmed again the inconsistency between the mass spectrum and decay widths of flavor partners in the octet-antidecuplet mixing scenario with $J^P = 1/2^+$. However, the assignment of $J^P = 3/2^-$ particles in the mixing scenario well reproduced the mass spectrum as well as the decay widths of $\Theta(1540)$, $N(1520)$, and $N(1700)$. Assignment of $3/2^-$ predicted a new Σ state at around 1840 MeV, and the nucleon mixing angle was close to the one of ideal mixing. The $1/2^-$ assignment was not realistic since the widths were too large for Θ^+ . In order to investigate the $3/2^+$ case, better established experimental data of the resonances were needed.

The assignment of $J^P = 3/2^-$ for exotic baryons seems reasonable also in a quark model especially when narrow width of the Θ^+ is to be explained [25]. The $(0s)^5$ configuration for the $3/2^- \Theta^+$ is dominated by the K^*N configuration [26], which however cannot be the decay channel due to the masses of K^* and N higher than the mass of Θ^+ . Hence we expect naturally (in addition to a naive suppression mechanism due to the d -wave KN decay) a strong suppression of the decay of the Θ^+ . The possibility of the spin $3/2$ for the Θ^+ or its excited states has been discussed not only in quark models [25, 26, 27, 28], but also in the KN potential model [29], the $K\Delta$ resonance model [30] and QCD sum rule calculations [31].

The $3/2^-$ resonances of nonexotic quantum numbers have been also studied in various models of hadrons. A conventional quark model description with a $1p$ excitation of a single quark orbit has been successful qualitatively [32]. Such three-quark states can couple with meson-baryon states which could be a source for the five- (or more-) quark content of the resonance. In the chiral unitary approach, $3/2^-$ states are generated by s -wave scattering states of an octet meson and a decuplet baryon [30, 33, 34]. By construction, the resulting resonances are largely dominated by five-quark content. These two approaches generate octet baryons which will eventually mix with the antidecuplet partners to generate the physical baryons. In other words, careful investigation of the octet states before mixing will provide further information.

In the present phenomenological study, we have found that $J^P = 3/2^-$ seems to fit observations to date. As we have known, other identifications have been also discussed in the literature, for instance, using large N_c expansion [35, 36, 37]. It is therefore important to determine the quantum numbers of Θ^+ in experiments [38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49], not

only for the exotic particles but also for the baryon spectroscopy of nonexotic particles. Study of high spin states in phenomenological models and calculations based on QCD are strongly encouraged [31].

Acknowledgments

We would like to thank Shi-Lin Zhu and Seung-il Nam for useful discussions. We acknowledge to Manuel J. Vicente Vacas and Daniel Cabrera for helpful comments. This work supported in part by the Grant for Scientific Research ((C) No.16540252) from the Ministry of Education, Culture, Science and Technology, Japan.

APPENDIX A: EXPERIMENTAL INFORMATION

In PDG [22], the masses and widths of Θ^+ and Ξ^{--} are given as

$$M_{\Theta^+} = 1539.2 \pm 1.6 \text{ MeV}, \quad \Gamma_{\Theta^+} = 0.9 \pm 0.3 \text{ MeV}. \quad (\text{A1})$$

$$M_{\Xi^{--}} = 1862 \pm 2 \text{ MeV}, \quad \Gamma_{\Xi^{--}} < 18 \text{ MeV}. \quad (\text{A2})$$

In Table VIII we summarize the resonances with several spins and parities. Note that the $\Sigma(1385)$ and the $\Xi(1530)$ are not listed because they are assigned in the decuplet with the $\Delta(1232)$.

APPENDIX B: MIXING ANGLE

By looking at the mass formulae given in subsection III A, the masses of mixed states can be written, in general, by

$$M_1(\theta) = A \cos^2 \theta + B \sin^2 \theta - \frac{(B-A)}{2} \tan 2\theta \sin 2\theta, \quad (\text{B1})$$

$$M_2(\theta) = A \sin^2 \theta + B \cos^2 \theta + \frac{(B-A)}{2} \tan 2\theta \sin 2\theta. \quad (\text{B2})$$

These functions obey the following relations

$$M_i(\theta) = M_i(\theta + \pi), \quad \text{for } i = 1, 2 \quad (\text{B3})$$

$$M_i(\theta) = M_i(\pi - \theta), \quad \text{for } i = 1, 2 \quad (\text{B4})$$

$$M_1(\theta) = M_2(\pi/2 - \theta), \quad M_2(\theta) = M_1(\pi/2 - \theta). \quad (\text{B5})$$

Equation (B3) shows that $M_1(\theta)$ and $M_2(\theta)$ are periodic functions with period π , while Eq. (B4) shows that there is a reflection symmetry of $0 \leq \theta \leq \pi/2$ and $\pi/2 \leq \theta \leq \pi$. In order to make a one to one correspondence between θ and the masses, the domain of θ should be $0 \leq \theta < \pi/2$. In addition, there is a discrete symmetry under the interchange $\theta \leftrightarrow \pi/2 - \theta$ and $M_1 \leftrightarrow M_2$, due to Eq. (B5). Fixing the assignment of M_1 and M_2 to the physical states, the mixing angle can be determined without duplication.

TABLE VIII: Resonances listed in PDG [22]. The $\Sigma(1385)$ and the $\Xi(1530)$ are not listed because they are assigned in the decuplet with the $\Delta(1232)$. We denote stars following the definition in PDG, except for three- or four-star resonances which are well established.

R	J^P	States
N^*	$1/2^-$	$N(1535)$, $N(1650)$, $N(2090)^*$
	$1/2^+$	$N(1440)$, $N(1710)$, $N(2100)^*$
	$3/2^+$	$N(1720)$, $N(1900)^{**}$
	$3/2^-$	$N(1520)$, $N(1700)$, $N(2080)^{**}$
Λ^*	$1/2^-$	$\Lambda(1405)$, $\Lambda(1670)$, $\Lambda(1800)$
	$1/2^+$	$\Lambda(1600)$, $\Lambda(1810)$
	$3/2^+$	$\Lambda(1890)$
	$3/2^-$	$\Lambda(1520)$, $\Lambda(1690)$, $\Lambda(2325)^*$
	unknown	$\Lambda(2000)^*$, $\Lambda(2585)^{**}$
Σ^*	$1/2^-$	$\Sigma(1620)^{**}$, $\Sigma(1750)$, $\Sigma(2090)^*$
	$1/2^+$	$\Sigma(1660)$, $\Sigma(1770)^*$, $\Sigma(1880)^{**}$
	$3/2^+$	$\Sigma(1840)^*$, $\Sigma(2080)^{**}$
	$3/2^-$	$\Sigma(1580)^{**}$, $\Sigma(1670)$, $\Sigma(1940)$
	unknown	$\Sigma(1480)^*$, $\Sigma(1560)^{**}$, $\Sigma(1690)^{**}$, $\Sigma(2250)$, $\Sigma(2455)^{**}$, $\Sigma(2620)^{**}$
Ξ^*	$1/2^-$	
	$1/2^+$	
	$3/2^+$	
	$3/2^-$	$\Xi(1820)$
	unknown	$\Xi(1620)^*$, $\Xi(1690)$, $\Xi(1950)$, $\Xi(2030)$, $\Xi(2120)^*$, $\Xi(2250)^{**}$, $\Xi(2370)^{**}$, $\Xi(2500)^*$

- [1] LEPS, T. Nakano *et al.*, Phys. Rev. Lett. **91**, 012002 (2003).
- [2] NA49, C. Alt *et al.*, Phys. Rev. Lett. **92**, 042003 (2004).
- [3] D. P. Roy, J. Phys. G **30**, R113 (2004).

- [4] S.-L. Zhu, Int. J. Mod. Phys. A **19**, 3439 (2004).
- [5] M. Oka, Prog. Theor. Phys. **112**, 1 (2004).
- [6] D. Diakonov, hep-ph/0406043.
- [7] R. L. Jaffe, hep-ph/0409065.

- [8] S. Sasaki, hep-lat/0410016.
- [9] K. Goeke, H.-C. Kim, M. Praszalowicz, and G.-S. Yang, hep-ph/0411195 [Prog. Part. Nucl. Phys. (to be published)].
- [10] J. J. de Swart, Rev. Mod. Phys. **35**, 916 (1963).
- [11] R. L. Jaffe and F. Wilczek, Phys. Rev. Lett. **91**, 232003 (2003).
- [12] T. D. Cohen, Phys. Rev. D **70**, 074023 (2004).
- [13] S. Pakvasa and M. Suzuki, Phys. Rev. D **70**, 036002 (2004).
- [14] M. Praszalowicz, Ann. Phys. (Berlin) **13**, 709 (2004).
- [15] A. Hosaka, T. Hyodo, F.J. Llanes-Estrada, E. Oset, J. R. Peláez, and M. J. Vicente Vacas, hep-ph/0411311 [Phys. Rev. C (to be published)].
- [16] P. Schweitzer, Eur. Phys. J. A **22**, 89 (2004).
- [17] D. Diakonov and V. Petrov, Phys. Rev. D **69**, 094011 (2004).
- [18] M. Praszalowicz, hep-ph/0410241.
- [19] J. R. Ellis, M. Karliner, and M. Praszalowicz, J. High Energy Phys. 05 (2004) 002.
- [20] D. Diakonov, V. Petrov, and M. V. Polyakov, Z. Phys. A **359**, 305 (1997).
- [21] S. H. Lee, H. Kim, and Y. Oh, hep-ph/0402135.
- [22] Particle Data Group, S. Eidelman *et al.*, Phys. Lett. B **592**, 1 (2004).
- [23] V. Guzey and M. V. Polyakov, hep-ph/0501010.
- [24] F. Stancu, Phys. Lett. B **595**, 269 (2004), Erratum-ibid, B **598**, 295 (2004)
- [25] A. Hosaka, M. Oka, and T. Shinozaki, hep-ph/0409102.
- [26] S. Takeuchi and K. Shimizu, hep-ph/0410286.
- [27] T. Inoue, V. E. Lyubovitskij, T. Gutsche, and A. Faessler, hep-ph/0407305.
- [28] Y. Kanada-En'yo, O. Morimatsu, and T. Nishikawa, hep-ph/0404144 [Phys. Rev. D (to be published)].
- [29] D. E. Kahana and S. H. Kahana, Phys. Rev. D **69**, 117502 (2004).
- [30] S. Sarkar, E. Oset, and M. J. V. Vacas, nucl-th/0404023.
- [31] T. Nishikawa, Y. Kanada-En'yo, Y. Kondo, and O. Morimatsu, hep-ph/0411224.
- [32] N. Isgur and G. Karl, Phys. Rev. D **18**, 4187 (1978).
- [33] S. Sarkar, E. Oset, and M. J. V. Vacas, nucl-th/0407025.
- [34] E. E. Kolomeitsev and M. F. M. Lutz, Phys. Lett. B **585**, 243 (2004).
- [35] E. Jenkins and A. V. Manohar, J. High Energy Phys. 06 (2004) 039.
- [36] M. E. Wessling, Phys. Lett. B **603**, 152 (2004).
- [37] D. Pirjol and C. Schat, Phys. Rev. D **71**, 036004 (2005).
- [38] T. Hyodo, A. Hosaka, and E. Oset, Phys. Lett. B **579**, 290 (2004).
- [39] K. Nakayama and K. Tsushima, Phys. Lett. B **583**, 269 (2004).
- [40] A. W. Thomas, K. Hicks, and A. Hosaka, Prog. Theor. Phys. **111**, 291 (2004).
- [41] Y.-s. Oh, H.-c. Kim, and S. H. Lee, Nucl. Phys. A **745**, 129 (2004).
- [42] C. Hanhart *et al.*, Phys. Lett. B **590**, 39 (2004).
- [43] S. I. Nam, A. Hosaka, and H. C. Kim, Phys. Lett. B **602**, 180 (2004).
- [44] Y. N. Uzikov, Phys. Lett. B **595**, 277 (2004).
- [45] K. Nakayama and W. G. Love, Phys. Rev. C **70**, 012201(R) (2004).
- [46] C. Hanhart, J. Haidenbauer, K. Nakayama, and U. G. Meissner, Phys. Lett. B **606**, 67 (2005).
- [47] W. Roberts, Phys. Rev. C **70**, 065201 (2004).
- [48] Y. N. Uzikov, nucl-th/0411113.
- [49] Y. Oh, K. Nakayama, and T. S. H. Lee, hep-ph/0412363.